The Speed and Mean Lifetime of Cosmic Ray Muons

Sean Condon*
MIT Department of Physics
(Dated: November 16, 2019)

Using two scintillation detectors, the velocity of cosmic ray muons at sea level was determined to be 30.14 ± 1.96 cm/ns, requiring a muon lifetime of at least 49.77 μs assuming formation above 15 km. By stopping muons in a plastic scintillator, the mean lifetime of muons at rest was determined to be 2.196 ± 0.055 μs. The discrepancy of muon lifetime is explained by relativistic time dilation and length contraction.

I. BACKGROUND THEORY

I.1. Muon Formation in Upper Atmosphere

Cosmic rays are bare atomic nuclei and solitary electrons that pervade the galaxy with an energy density of around 1 eV/cm^3, and can have energies up to 10^{21} eV [7]. When energetic cosmic rays collide with air molecules in Earth's upper atmosphere, a cascade of particles and electromagnetic radiation called an air shower occurs. In this cascade, charged pions ($\pi^{\pm}$) created primarily in the initial interaction of cosmic ray and air nucleus, decay into muons ($\mu^{\pm}$) and neutrinos (ν) according to equation 1 [5]:

$$\pi^{\pm} \rightarrow \mu^{\pm} + \nu \tag{1}$$

High altitude observations show that most muons from cosmic ray collisions form at altitudes above 15 km [7]. Experimental observation also shows that the intensity I of penetrating particles, like the muon, has the following relationship on the zenith angle ϕ [7]:

$$I(\phi) = I_{\mu} \cos^2(\phi) \tag{2}$$

where $I_{\mu} = 0.83\,\text{x}10^{-2}\text{cm}^{-2}\text{s}^{-1}\text{str}^{-1}$ [7]. This equation gives an approximate muon flux at sea level of 1 cm^{-2}min^{-1}[1].

I.2. Special Relativity for Particle Velocities

In Newtonian physics, where the Galilean Transformation governs relationships between reference frames, relative velocities can add to infinity [7]. Given the high velocity of muons, Einstein's special theory of relativity provides a more reliable mathematical framework. This theory relates a particle's velocity to its momentum p, mass m, and the speed of light c with [2]:

$$v = \sqrt{\frac{(p/m)^2}{1 + \frac{(p/m)^2}{c^2}}} \tag{3}$$

Most muons at sea level have momentums between $p =$ 0.7 - 10GeV/c, and thus have velocities between 0.989 - 0.99994c [8].

Special relativity also introduces changes in relative length and time between earth's stationary reference frame E and a particle's stationary reference frame P:

$$L_P = \frac{L_E}{\gamma} \qquad T_E = \gamma T_P \tag{4}$$

where $\gamma = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$ is the Lorentz factor of particle velocity v; L and T are arbitrary distances and times. These relations are length contraction and time dilation.

II. PROCEDURES AND APPARATUS

II.1. Time of Flight Measurement with Scintillators

Once cosmic ray muons penetrate down to sea level, they are detected using scintillators. Scintillation detectors emit photons when struck by muons, and the photons propagate through the material until they reach a Photomultiplier Tube (PMT) at one end of the device [7]. It is important to note that the energy deposited in a scintillator is proportional to $\cos(\phi)^{-1}$ for zenith angle ϕ [4].

The PMT contains a photocathode to convert the photons into an electrical signal, which is amplified by a series of dynodes across high voltages upwards of 1.5 kV. Signal amplitudes from the PMT that are above some variable threshold on a constant fraction discriminator (CFD) are counted as muon detection events.

The velocity of sea level muons can be determined by analyzing the delay of muon detections between an upper scintillator and a lower scintillator at known separation. The PMT voltages and the CFD cutoffs for the upper and lower scintillator are adjusted to obtain about 190 and 140 counts per second. Given both scintillators are 40 cm x 60 cm, we expect a flux of 40 muons per second.

The time delay between pulses from the top and bottom scintillator is converted into a voltage using a time amplitude converter (TAC), and the resulting voltage is histogrammed into one of 2048 bins with a multi-channel analyzer (MCA). This delay is called the *time of flight*.

---

* scondon@mit.edu

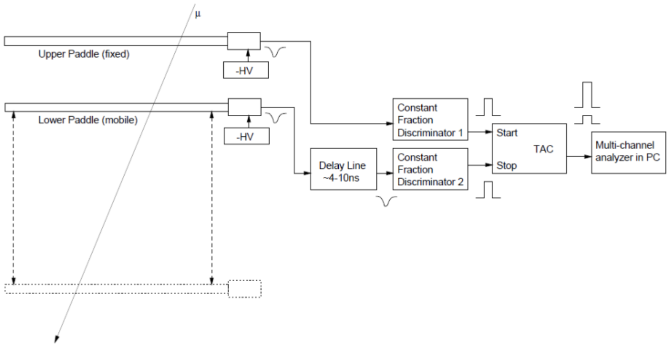

FIG. 1. Signal chain used to measure average speed of sea level muons. The upper and lower paddles are the scintillation detectors, and the boxes to their right with HV are the PMTs.

The signal chain of the lower scintillator contains a delay line of 4.88 meters of extra BNC cable to ensure the lower pulse always arrives at the TAC after the upper pulse. Given that the TAC searches for coincidence within a range of 100 ns, we expect 9.5 accidental coincidences per hour with our scintillator count rates. Because our longest run had over 3000 coincidences in 1.5 hours, accidental coincidences are a negligible component of the time of flight data.

The full signal chain used to measure average times of flight for muons over a specific scintillator separation is shown in figure 1.

II.2. Measurement of Muon Mean Lifetime

To determine the mean lifetime of muons, a signal chain capable of distinguishing muon lifetime events from background, and then measuring time differences between these events was needed.

The signal chain begins in a large block of plastic scintillator in a dark container. When muons come to rest in the scintillator, they deposit an energy of ~50 MeV. When muons at rest undergo decay in the scintillator, their electrons deposit an energy of ~20 MeV.

When one of these events occurs, the scintillation is converted into an amplified electrical pulse with a PMT. These pulses are filtered through a CFD to distinguish events from background. Our signal chain consists of two scintillators, two PMTs, and two CFDs. If a coincidence circuit determines that the output of the two CFDs both depict the same event, the coincidence circuit output is split and fed into the TAC's START and STOP inputs.

To prevent a single muon event from triggering both the TAC START and STOP, we delay the START input with a fixed length of BNC cable. This way, a muon coming to rest in the scintillator triggers the TAC to start through the delayed line, and then its decay triggers the TAC to stop through its non-delayed line. The delay introduced by the delay line should be much shorter than the anticipated time interval the experiment will observe.

The time between a muon coming to rest and a muon decaying is recorded by the TAC as some voltage, and then histogrammed with an MCA into 2048 different channels. The calibration between MCA voltages and time intervals is described in the next section. We recorded over 7,500 time differences over a period of around 16 hours.

II.3. MCA Calibration

In both the muon velocity and muon decay procedures, the final form of data is a histogram of voltages on an MCA. In both cases, we need to convert the histogram of voltages to a histogram of time differences. This is done by calibrating the MCA with signal pulses of precisely known time intervals using the Time Calibrator. The MCA was calibrated once for the muon velocity measurement, and once for the mean lifetime measurement, each using Time Calibrator signals with period slightly greater than the maximum expected result for that experiment.

III. RESULTS AND ANALYSIS

III.1. Mean Velocity of Cosmic Ray Muons

Using the setup described in section II.1, time of flight histograms were recorded at five different scintillator separations over two days. The average time of flight ΔT_{avg} of each histogram is interpreted as:

$$\Delta T_{\text{avg}} = \frac{D_{\text{avg}}}{v_\mu} + \Phi_{\text{system}} \tag{5}$$

where D_{avg} is the mean path length for that scintillator separation, v_μ is the muon velocity, and Φ_{system} is the *system lag*. Because we are using a delay line on the lower scintillator, and pulses take time to travel throughout the BNC cable, the time delay measured by the TAC is the muon time of flight D_{avg}/v_μ plus the time delay caused by differences in cabling Φ_{system}. The *system lag* is a constant of the setup, so if we have multiple measurements with the same setup, we can subtract out this constant value.

An example MCA histogram for one scintillator separation is shown in figure 2. The average time of flight ΔT is the mean of the data, and its uncertainty is the standard error of the mean.

Because incident muons can take any path between the scintillators, a Monte Carlo algorithm was used to determine the average distance D_{avg} for each detector separation. Given the muon flux angular dependence of equation 2, and the fact that energy deposited in the scintillator is proportional to $\cos(\phi)^{-1}$ for zenith angle ϕ, certain muon trajectories are more likely than others.

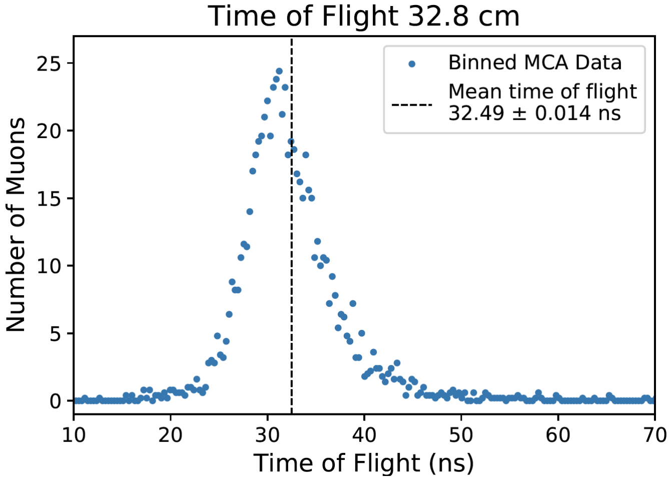

FIG. 2. Raw MCA data for muon time of flight measurement. Each data point is the average of 5 MCA channels.

Thus we compute D_{avg} with a Weighted Monte Carlo algorithm for 1 million random simulated positions on the upper and lower scintillator, each of which has weight w_i:

$$w_i = \cos^2(\phi_i) * \frac{1}{\cos(\phi_i)} \tag{6}$$

where ϕ_i is the zenith angle of the i-th trial. Then D_{avg} is the weighted average of euclidean distance across all one million trials. This Monte Carlo Simulation was implemented by Danny Ben-David in 2013, and the open source code is available online [4].

We analyzed the three separations from day one and the two separations from day two by fitting each with an independent linear relationship. Both days showed similar slopes (D_{avg}/v_μ), but had different x-intercepts (Φ_{system}). The difference in *system lag* between the two days $\Delta\Phi_{\text{system}}$ was 1.392 ns. It is very likely this systematic difference was caused by using slightly different BNC cables between the two days. $\Delta\Phi_{\text{system}}$ can be caused by a difference of only 28.3 cm of BNC cabling [3]. To correct for this, we subtract this time difference from the second day times of flight.

The final plot of time of flight versus mean flight distance is shown in figure 3. The best fit linear relationship for the data has a probability of fit of 71.8%, and gives the velocity of a muon to be 30.14 ± 1.96 cm/ns.

III.2. Mean Lifetime of Cosmic Ray Muons

Over a period of 16 hours, 7,500 events interpreted as muon lifetimes were recorded according to the procedures outlined in section II.2. The data, a histogram with 2048 channels, was rebinned: six channels were combined into one. The uncertainty on each bin is taken as the square root of the number of counts in that bin.

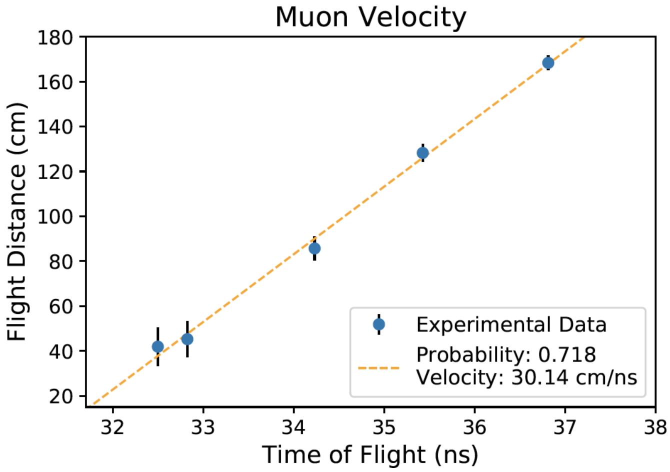

FIG. 3. Muon time of flight versus mean flight distance. Linear fit gives muon velocity of 30.14 ± 1.96 cm/ns with probability of fit of 71.8%.

The rebinned data is then fit with an exponential decay equation on top of some background noise. The functional form of this fit is shown in equation 7:

$$N_\mu = ae^{\frac{-t}{\tau}} + b \tag{7}$$

where N_μ gives the number of muons at time t, a is some constant, b is the offset to account for non-exponential background effects, and τ is the mean lifetime of cosmic ray muons at rest. Using the fit shown in figure 4, the mean lifetime of muons at rest is:

$$\tau = 2.196 \ \pm \ 0.055 \ \mu\text{s} \tag{8}$$

IV. DISCUSSION AND ERROR ANALYSIS

IV.1. Uncertainty on Muon Velocity

In section III.1, the velocity of the muon is reported as 30.14 ± 1.96 cm/ns. Apart from the statistical errors on this value - the standard error of the mean of time of flight, and the standard deviation of mean flight distance - there are several systematic errors to address.

As discussed in the introductory theory, cosmic ray muons at sea level can have momentum as low as 0.7 GeV, and as high as many GeVs [8]. This produces a dispersion in times of flight of 0.036 ns / m. At our smallest scintillator separation, this contributes an uncertainty of 0.1%, so the effect is negligible.

Additionally, muon scintillations take time to travel to the PMT, so each traversal event has some overestimation or underestimation on the velocity of the muon, depending on the relative position of the events on the upper and lower scintillators. Because overestimations

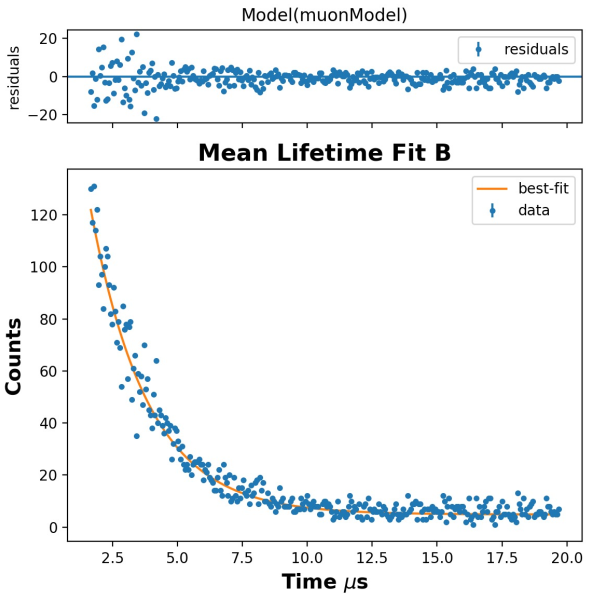

FIG. 4. The observed number of counts at various time intervals, fit with an exponential decay function according to equation 7. The probability of fit is 33%.

and underestimations are equally likely, this effect causes a Monte Carlo simulated net time dispersion of less than 0.01 ns over 3000 simulated trials.

A full listing of uncertainty sources is given in table I.

Source of Uncertainty	Size
Monte Carlo simulated path lengths	.052
Scintillator separation measurements	.013
Muon momentum dispersion	$< .001$
MCA calibration, standard error time of flight, accidental coincidence, scintillator travel time	negligible

TABLE I. Notable sources of uncertainty in muon time of flight measurements

IV.2. Muon Mean Lifetime Uncertainties

A full listing of the significant errors in the measurement of the mean lifetime of a muon can be found in table II.

IV.3. Special Relativity and Muon Lifetimes

At the measured velocity of 30.14 cm/ns, assuming formation above an altitude of 15 km, cosmic ray muons must survive for at least 49.77 μs to be observed at sea level.

Source of Uncertainty	Size
Fit parameter τ_μ	.025
Fit parameter b	.042
MCA Calibration	$< .001$
7.7% nuclear capture of μ^- by Carbon [6]	-.035

TABLE II. Notable sources of uncertainty in muon mean lifetime measurement

If we assume that muons have the same lifetime in flight as they do at rest, then the probability of a muon reaching sea level from an altitude of 15 km can be estimates as $e^{-50/2.18} = 1.09 \text{ x } 10^{-10}$.

In order to observe the one muon per square centimeter per minute that we observe at sea level, the muon flux in the upper atmosphere must be much higher than has been experimentally shown [7].

This contradiction is strong evidence from the theory of special relativity. According to this theory, high-speed muons make it to sea level before decaying because, in the particle's reference frame, the length to sea level is contracted by equation 4, and in the laboratory's perspective, the muons's time frame gets dilated according to equation 4.

[1] Muons. https://cosmic.lbl.gov/SKliewer/Cosmic_Rays/Muons.htm.

[2] Solving for the velocity of a particle, given a relativistic momentum. http://spiff.rit.edu/classes/phys200/lectures/mom_rel/isolate_v.html.

[3] Speed of Signal through Coaxial Cable. UCSB Physics.

[4] Danny Ben-David. https://jlab.mit.edu/wiki/14._The_Speed_and_Mean_Life_of_Cosmic-Ray_Muons.

[5] Badanaval Venkata Sreekantan M. V. S. Rao. Extensive Air Showers, 1998.

[6] D.F. Measday. The nuclear physics of muon capture, 2000.

[7] MIT Department of Physics. The Speed and Decay of Cosmic-Ray Muons: Experiments in the Relativistic Kinematics of the Universal Speed Limit and Time Dilation, 2017.

[8] Sundaresh Sankrith Prashant Shukla. Energy and angular distributions of atmospheric muons at the Earth, 2018.